Solving Systems of Equations

Elimination To solve a system of linear equations by elimination, add or subtract the equations to eliminate one of the variables. You may first need to multiply one or both of the equations by a constant so that one of the variables has the opposite coefficient in one equation as it has in the other.

Example 1 Use the elimination method to solve the system of equations.

\[ 2x - 4y = -26 \]
\[ 3x - y = -24 \]

Multiply the second equation by -4. Then add the equations to eliminate the \( y \) variable.

\[
\begin{align*}
2x - 4y &= -26 \\
3x - y &= -24 \\
\text{Multiply by } -4. & & 2x - 4y = -26 \\
& & 12x + 4y = 96 \\
& & -10x = 70 \\
& & x = -7 \\
\end{align*}
\]

Replace \( x \) with -7 and solve for \( y \).

\[
\begin{align*}
2x - 4y &= -26 \\
2(-7) - 4y &= -26 \\
-14 - 4y &= -26 \\
-4y &= -12 \\
y &= 3
\end{align*}
\]

The solution is (-7, 3).

Example 2 Use the elimination method to solve the system of equations.

\[ 3x - 2y = 4 \]
\[ 5x + 3y = -25 \]

Multiply the first equation by 3 and the second equation by 2. Then add the equations to eliminate the \( y \) variable.

\[
\begin{align*}
3x - 2y &= 4 \\
5x + 3y &= -25 \\
\text{Multiply by } 3. & & 9x - 6y = 12 \\
\text{Multiply by } 2. & & 10x + 6y = -50 \\
& & 19x = -38 \\
& & x = -2
\end{align*}
\]

Replace \( x \) with -2 and solve for \( y \).

\[
\begin{align*}
3x - 2y &= 4 \\
3(-2) - 2y &= 4 \\
-6 - 2y &= 4 \\
-2y &= 10 \\
y &= -5
\end{align*}
\]

The solution is (-2, -5).
Exercises

Solve each system of equations by using elimination.

1. \[2x - y = 7\]
   \[3x + y = 8\]

2. \[x - 2y = 4\]
   \[-x + 6y = 12\]

3. \[3x + 4y = -10\]
   \[x - 4y = 2\]

4. \[3x - y = 12\]
   \[5x + 2y = 20\]

5. \[4x - y = 6\]
   \[2x - \frac{y}{2} = 4\]

6. \[5x + 2y = 12\]
   \[-6x - 2y = -14\]

7. \[2x + y = 8\]
   \[3x + \frac{3}{2}y = 12\]

8. \[7x + 2y = -1\]
   \[4x - 3y = -13\]

9. \[3x + 8y = -6\]
   \[x - y = 9\]

10. \[5x + 4y = 12\]
    \[7x - 6y = 40\]

11. \[-4x + y = -12\]
    \[4x + 2y = 6\]

12. \[5m + 2n = -8\]
    \[4m + 3n = 2\]